Short Notes

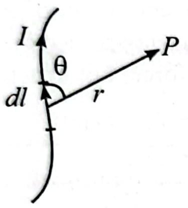
A static charge produces only electric field. A moving charge produces both electric field and magnetic field. A current carrying conductor produces only magnetic field.

Magnetic Field Produced by a Current (Biot-Savart Law)

The magnetic induction dB produced by an element dl carrying a current I at a distance r is given by:

$$dB = \frac{\mu_0\mu_r}{4\pi}\frac{I\,dl\sin\theta}{r^2} \Rightarrow \vec{dB} = \frac{\mu_0\mu_r}{4\pi}\frac{I(\vec{dl}\times\vec{r})}{r^3}$$

Here, the quantity Idl is called as current element.



μ = permeability of the medium = $\mu_0\mu_r$

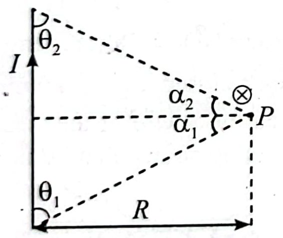
μ_0 = permeability of free space

μ_r = relative permeability of the medium (Dimensionless quantity)

Unit of μ_0 and μ is N A^{-2} or H m^{-1}; $\mu_0 = 4\pi \times 10^{-7}$ H m^{-1}

Magnetic Induction Due to a Straight Current Conductor

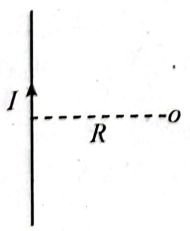
(i) Magnetic induction due to a finite wire.



$$B = \frac{\mu_0 I}{4\pi R}(\cos\theta_1 + \cos\theta_2) = \frac{\mu_0 I}{4\pi R}(\sin\alpha_1 + \sin\alpha_2)$$

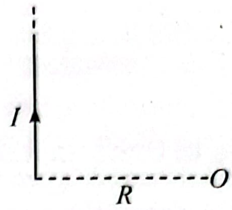
(ii) Magnetic induction due to a infinitely long wire

$$B = \frac{\mu_0 I}{2\pi R}\otimes\ (\alpha_1 = 90°;\ \alpha_2 = 90°)$$

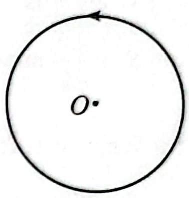


(iii) Magnetic induction due to semi infinite straight conductor

$$B = \frac{\mu_0 I}{4\pi R}\otimes\ (\alpha_1 = 0°;\ \alpha_2 = 90°)$$



Magnetic Field Due to a Flat Circular Coil Carrying a Current



(i) At its centre: $B = \frac{\mu_0 NI}{2R}\odot$

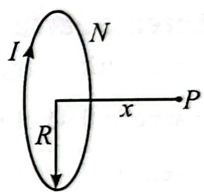
where

N = total number of turns in the coil

I = current in the coil

R = Radius of the coil

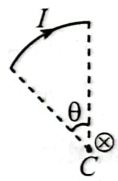
(ii) On the axis: $B = \frac{\mu_0 NIR^2}{2(x^2+R^2)^{3/2}}$



Where x = distance of the point from the centre.

It is maximum at the centre, $B_C = \frac{\mu_0 NI}{2R}$

(iii) Magnetic field due to flat circular arc :



$$B = \frac{\mu_0 I\theta}{4\pi R}$$

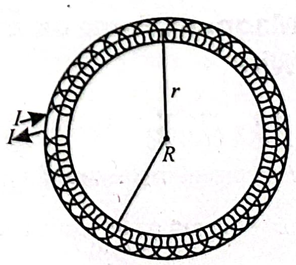
Magnetic Field Due to Infinite Long Solid Cylindrical Conductor of Radius R

- For $r \geq R : B = \frac{\mu_0 I}{2\pi r}$
- For $r < R : B = \frac{\mu_0 Ir}{2\pi R^2}$

Magnetic Induction Due to a Solenoid

$B = \mu_0 nI$, where n is number of turns per meter and I is current. Direction is along the axis.

Magnetic Induction Due to Toroid



$B = \mu_0 nI$

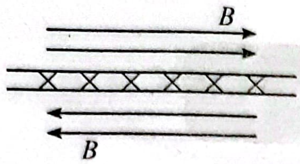
Where $n = \frac{N}{2\pi R}$

R is radius of toroid

N is total turns and $R \gg r$

Magnetic Induction Due to Current Carrying Sheet

$B = \frac{1}{2}\mu_0\lambda$ (λ = Linear current density (A/m))



Ampere's Circuital Law

$\oint \vec{B}\cdot d\vec{l} = \mu_0 \Sigma I$ where ΣI = algebraic sum of all the enclosed current.

Motion of a Charge In Uniform Magnetic Field

(*a*) When $\vec{v}$ is || to $\vec{B}$: Motion will be in a straight line and $\vec{F} = 0$

(*b*) When $\vec{v}$ is ⊥ to $\vec{B}$: Motion will be in circular path with radius $R = \dfrac{mv}{qB}$ and angular velocity $\omega = \dfrac{qB}{m}$ and $F = qvB$.

(c) When $\vec{v}$ is at angle θ to $\vec{B}$: Motion will be helical with radius $R = \dfrac{mv\sin\theta}{qB}$ and pitch $P_H = \dfrac{2\pi mv\cos\theta}{qB}$ and $F = qvB\sin\theta$.

Lorentz Force

An electric charge 'q' moving with a velocity $\vec{v}$ through a magnetic field of magnetic induction $\vec{B}$ experiences a force $\vec{F}$, given by $\vec{F} = q\vec{v}\times\vec{B}$. There fore, if the charge moves in a space where both electric and magnetic fields are superposed.

$\vec{F}$ = net electromagnetic force on the charge = $q\vec{E} + q\vec{v}\times\vec{B}$

This force is called the Lorentz Force

Motion of Charge in Combined Electric Field and Magnetic Field

- When $\vec{v} \parallel \vec{B}$ and $\vec{v} \parallel \vec{E}$, motion will be uniformly accelerated in line as $F_{magnetic} = 0$ and $F_{electrostatic} = qE$

  So the particle will be either speeding up or speeding down
- When $\vec{v} \parallel \vec{B}$ and $\vec{v} \perp \vec{E}$, motion will be uniformly accelerated in a parabolic path
- When $\vec{v} \perp \vec{B}$ and $\vec{v} \perp \vec{E}$, the particle will move undeflected and undeviated with same uniform speed if $v = \dfrac{E}{B}$ (This is called as velocity selector condition)

Magnetic Force on a Straight Current Carrying Wire

$\vec{F} = I(\vec{L}\times\vec{B})$

I = current in the straight conductor

$\vec{L}$ = length of the conductor in the direction of the current in it

$\vec{B}$ = magnetic induction (Uniform throughout the length of conductor)

Note: In general force is $\vec{F} = \int I(d\vec{l}\times\vec{B})$

Magnetic Interaction Force Between Two Parallel Long Straight Currents

The interaction force between 2 parallel long straight wires is:

(*i*) Repulsive if the currents are anti-parallel.

(*ii*) Attractive if the currents are parallel.

This force per unit length on either conductor is given by $F = \dfrac{\mu_0}{2\pi}\dfrac{I_1 I_2}{r}$.

Where r = perpendicular distance between the parallel conductors

Magnetic Torque on a Closed Current Circuit

When a plane closed current circuit of 'N' turns and of area 'A' per turn carrying a current I is placed in uniform magnetic field, it experience a zero net force, but experience a torque given by

$\vec{\tau} = NI\vec{A}\times\vec{B} = \vec{M}\times\vec{B} = BINA\sin\theta$

where $\vec{A}$ = area vector outward from the face of the circuit where the current is anticlockwise, $\vec{B}$ = magnetic induction of the uniform magnetic field and

$\vec{M}$ = magnetic moment of the current circuit = $NI\vec{A}$.

Force on a Random Shaped Conductor in a Uniform Magnetic Field

- Magnetic force on a closed loop in a uniform $\vec{B}$ is zero
- Force experienced by a wire of any shape is equivalent to force on a wire joining points A and B in a uniform magnetic field.

Magnetic Moment of A Rotating Charge

If a charge q is rotating at an angular velocity ω, its equivalent current is given as $I = \dfrac{q\omega}{2\pi}$ and its magnetic moment is $M = I\pi R^2 = \dfrac{1}{2}q\omega R^2$.